

1.4 Solve Absolute Value Equations

Absolute Value is the distance from zero.

$|5| = 5$

$|16| = 16$

$|x| = 10$ means $x = ?$ 10 or -10

$|-5| = 5$

$|-16| = 16$

check
 $|10| = 10$ $|-10| = 10$

two solutions with x

Important tips to remember when solving an Absolute Value Equation:

1. Treat the absolute value bars as a grouping symbol... **BUT NEVER Distribute** into them!!!
2. **Isolate** the absolute value bars to solve.
3. Remember: the value inside can be positive or negative. **2 cases!!!**
4. **Check** final answers for **Extraneous Values!!!** After substituting the value in for the variable **EVALUATE (apply)** the absolute value.

Examples:

<p>1. $x+6 = 9$</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> $x+6 = 9$ $x = 3$ </div> <div style="text-align: center;"> $x+6 = -9$ $x = -15$ </div> </div> <p>check</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> $3+6 = 9$ $9 = 9$ $9 = 9 \checkmark$ </div> <div style="text-align: center;"> $-15+6 = 9$ $-9 = 9$ $9 = 9 \checkmark$ </div> </div>	<p>2. $\frac{4}{4} 3x-5 = \frac{8x}{4}$ isolate </p> <p>$3x-5 = 2x$</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> $3x-5 = 2x$ $-5 = -x$ $5 = x$ </div> <div style="text-align: center;"> $3x-5 = -2x$ $-5 = -5x$ $1 = x$ </div> </div> <p>check</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> $4 3(5)-5 = 8(5)$ $4 15-5 = 40$ $4 10 = 40$ $40 = 40 \checkmark$ </div> <div style="text-align: center;"> $4 3(1)-5 = 8(1)$ $4 3-5 = 8$ $4 -2 = 8$ $8 = 8 \checkmark$ </div> </div>
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3. $|x-3|+7=2$
 $-7 -7$

$|x-3| = -5$
 \uparrow no solution

an absolute value cannot = negative before separation into 2 equations

$x-3 = -5$ $x-3 = 5$
 $x = -2$ $x = 8$

check

$ -2-3 +7=2$	$ 8-3 +7=2$
$ 5 +7=2$	$15 +7=2$
$5+7 \neq 2$	$12 \neq 2$

4. $\frac{1}{2}|x-2|+3=x-2$
 $-3 -3$

$\frac{1}{2}|x-2| = x-5$ mult. by 2
 $\frac{1}{2}$ $\frac{1}{2}$

$|x-2| = 2x-10$

$x-2 = 2x-10$
 $-x -x$
 $-2 = x-10$
 $+10 +10$
 $8 = x$

check
 $\frac{1}{2}|8-2|+3=8-2$
 $\frac{1}{2} \cdot 6$
 $3+3 = 8-2$
 $6 = 6 \checkmark$

$x-2 = -2x+10$
 $+2x +2x$
 $3x-2 = 10$
 $+2 +2$
 $3x = 12$
 ~~$x = 4$~~

check
 $\frac{1}{2}|4-2|+3=4-2$
 $1+3 = 2$
 $4 \neq 2$
extraneous

5. $|\frac{x}{3}-\frac{2}{3}|=1$

$\frac{x}{3}-\frac{2}{3}=1$

mult by 3

$x-2=3$
 $x=5$

ck
 $|\frac{5}{3}-\frac{2}{3}|=1$
 $|\frac{3}{3}|=1$
 $1=1 \checkmark$

$\frac{x}{3}-\frac{2}{3}=-1$
 $x-2=-3$
 ~~$x=-1$~~

$|\frac{-1}{3}-\frac{2}{3}|=-1$
 $|\frac{-3}{3}|=-1$
 $1 \neq -1$
 extraneous

6. $|3z+1|=-6z$

$3z+1 = -6z$
 $-3z -3z$
 $1 = -9z$
 $\frac{1}{-9} \frac{-9}{-9}$
 $-\frac{1}{9} = z$

check
 $|3(-\frac{1}{9})+1| = -6(-\frac{1}{9})$
 $|-1/3+1| = 2/3$
 $|\frac{2}{3}| = 2/3$
 \checkmark



$3z+1 = 6z$
 $-3z -3z$
 $1 = 3z$
 ~~$\frac{1}{3} = z$~~

$|3(\frac{1}{3})+1| = -6(\frac{1}{3})$
 $|1+1| = -2$
 $|2| = -2$
 $2 \neq -2$

~~116~~ Solve Absolute Value Inequalities

You can solve an absolute value inequality by rewriting it as a compound inequality and solving each part.

GRAPH:


$ x < 4$ intersection This means the distance between x and 0 is less than 4 units.  $-4 < x < 4$	$ x > 4$ UNION This means the distance between x and 0 is greater than 4 units.  $x < -4$ or $x > 4$
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In conclusion, to know what type of compound inequality to use, follow these rules:

AND is represented by: $ x <$ $_ < x < _$ less than (must intersect)	OR is represented by: $ x >$ $x > _$ or $x < _$ greater than
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Examples:

1. $|x-2| \leq 8$ *keep same* *reverse inequality and change sign*

$x-2 \leq 8$ $x \leq 10$	$x-2 \geq -8$ $x \geq -6$	1. $-6 \leq x \leq 10$ 
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OR

2. $\frac{3|x+4|}{3} > \frac{18}{3}$

$ x+4 > 6$ $x+4 > 6$ $x > 2$	$x+4 < -6$ $x < -10$	2. $x < -10$ or $x > 2$ 
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or

$$3. \quad 7 + |9 - 5x| > 1$$

$$\quad \quad \quad -7 \quad \quad \quad -7$$

$$|9 - 5x| > -6$$

$$9 - 5x > -6$$

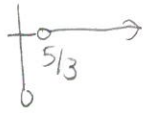
$$-5x > -15$$

$$x < 3$$

$$9 - 5x < 6$$

$$3 < 5x$$

$$\frac{5}{3} < x$$



these overlap
so all real numbers

$$3. \quad \underline{x < 3 \text{ or } x > \frac{5}{3}}$$



$$4. \quad |4x| + 3 \leq 0$$

$$|4x| \leq -3$$

$$4x \leq -3 \quad 4x \geq 3$$

$$x \leq -\frac{3}{4} \quad \text{and} \quad x \geq \frac{3}{4}$$

←

→

no intersection

check a value to be sure

$$4. \quad \underline{\text{no solution}}$$



$$5. \quad |x - 3| < 2x + 3$$

$$x - 3 < 2x + 3$$

$$-3 < x + 3$$

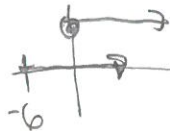
$$-6 < x$$

$$x - 3 > -2x - 3$$

$$3x - 3 > -3$$

$$3x > 0$$

$$x > 0$$



overlaps at $x > 0$

$$5. \quad \underline{x > 0}$$

